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Research Article

HFMD- A Fuzzy Game Theory Model

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Abstract

In this project, we developed a fuzzy game theory modelforHand Foot Mouth disease (HFMD), a transmissible disease affecting many populations around the world. HFMD is fomented from the genus enteroviruses. Although there are several viruses, coxsackie virus enteroviruses 71 (EV71) and A16 [5]are most probable to produce this disease. Precise estimates of disease transmission rate are critical for epidemiological Game theory model. We considered the transmission co-efficient to be uncertain and described by a fuzzy number which is triangular and symmetric. We defuzzified the fuzzy number by a signed distance method and studied sensitivity analysis.

Keywords: Game theory, Nash equilibrium, Triangular fuzzy number.

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1. Introduction

Hand Foot Mouth Disease (HFMD) is a transmissible infection affecting many populations around the world. HFMD can be found in many countries [4, 6] and is most common in infants and children, particularly those who are 10 years and younger, with emphasis on the age group zero to five. Although it is most common amongst youth, HFMD can also betide adults.

HFMD is generally transmitted by oral ingestion. Some examples of this include mucus, saliva, faces of infected persons, nasal secretions, stool, fluid from blisters, coughs, and sneezes (respiratory droplets). Common symptoms of HFMD include a fever, vomiting, poor appetite, painful sores, headache, fatigue, sore throat, skin blister-like lesions, dehydration, irritability, and sometimes asymptomatic [1].

Roughly, this disease lasts for 10 days and normally occurs in the spring, summer, or fall. Currently, there are no specific treatments, but there are treatments to help with individual symptoms.

For this reason, the best ways to prevent this disease are cleansing shared spaces, hands, and isolation from those infected. One protective gear that can be used to ensure better prevention is a medical face mask. This is used to help reduce the spread of saliva.

In summary, Hand Foot Mouth Disease generally affects infants and children five years and younger. There is currently no vaccine to counter HFMD. A medical mask can be worn to further prevent spreading. Although the disease may last up to 10 days, the virus may stay in the body for weeks. There is currently no vaccine in the US to protect against the viruses that cause HFMD. The best way to prevent the infection is through good hygiene.

2. Model

We modified the epidemiological model of Karaket et al [2], and relied on the results and analysis, of that modified version of the epidemiological model. Fig-1 is the modified version of the transmission dynamics and Table-1 has explained the notations of the parameters. Fig-2 represents the triangular fuzzy number [3].

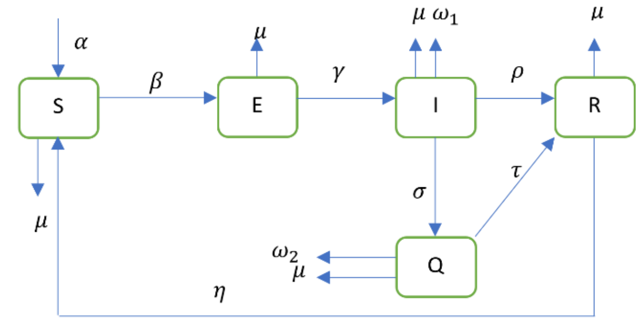

Figure 1: Transmission Dynamics

In this model transmission co-efficient (symptomatic) γ is uncertain and we describe it by triangular fuzzy number which is symmetric.

Table1: Parameter Description

Parameter	Biological meaning
α	Birth Rate
β	Transmission co-efficient (asymptomatic)
γ	Transmission co-efficient (symptomatic)
σ	Quarantine rate
ρ	Recovery rate of the infected
τ	Recovery rate of the quarantined
η	Rate from recovered to susceptible
μ	Natural death rate
ω_1	Infected disease-relate death rate
ω_2	Quarantined disease-relate death rate
g	Proportion of mask uses

3. Results

Force of Infection $\lambda(g,g_p) = (1-g)\beta I$

Where g is the proportion of mask usages $\in [0,1]$and β is transmission co-efficient and g_p is the population using mask.

$\lambda(g,g_p) = (1-g)\beta I = (1-g)\lambda(0,g_p)$

$\lambda(0,g_p) = \beta I$

$\lambda'(g,g_p) = -\lambda(o,g_p)$

Payoff function

$$E(g,g_p) = -Cg - \frac{\tilde{\gamma}\lambda(g,g_p)}{(\tilde{\gamma}+\mu)(\lambda(g,g_p)+\mu)}$$

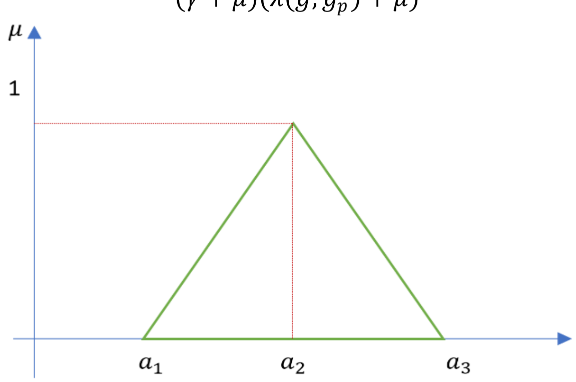

Fig-2: Triangular Fuzzy Number

The transmission co-efficient is$\tilde{\gamma} = (\gamma-\Delta, \Delta, \gamma+\Delta)$. To defuzzify the transmission co-efficient we introduce the signed distance method [3]. As $\tilde{\gamma} \in ℶ$ where ℶ be the family of all fuzzy sets $\tilde{\gamma}$ defined on $\mathbb{R}$.For$\tilde{\gamma} \in ℶ$, the signed distance is

$$\frac{1}{2}\int_0^1 [(1/\gamma)_L(\alpha) + (1/\gamma)_U(\alpha)]d\alpha$$

$$= \frac{\mu}{2\Delta} ln \frac{\gamma+\Delta}{\gamma-\Delta}$$

The defuzzified Payoff function is

$$E(g,g_p) = Cg - \frac{(1-g)\lambda(0,g_p)}{(1-g)\lambda(0,g_p)+\mu}\left[1 - \frac{\mu}{2\Delta} ln \frac{\gamma+\Delta}{\gamma-\Delta}\right], \left|\frac{\Delta}{\gamma}\right| < 1$$

$$\frac{dE(g)}{dg} = -C + \frac{\mu\lambda(0,\ g_p)}{1-g)\lambda(0,g_p)+\mu]^2}\left[1 - \frac{\mu}{2\Delta} ln \frac{\gamma+\Delta}{\gamma-\Delta}\right], \left|\frac{\Delta}{\gamma}\right| < 1$$

$$\frac{d^2E(g)}{dg^2} = \frac{2\mu\lambda^2(0,g_p)}{[(1-g)\lambda(0,g_p)+\mu]^3}\left[1 - \frac{\mu}{2\Delta} ln \frac{\gamma+\Delta}{\gamma-\Delta} > 0 \ if \ \frac{\mu}{\gamma} < 1\right.$$

as

$$\left[1 - \frac{\mu}{2\Delta} \ln \frac{\gamma+\Delta}{\gamma-\Delta} = 1 - \frac{\mu}{\gamma}\left[1 + \frac{\Delta^2}{3\gamma^2} + \frac{\Delta^4}{5\gamma^4} + \cdots\right], \frac{\Delta}{\gamma} < 1\right.$$

Our Nash Equilibrium is

$E(0,g_p)$=$E(1,g_p)$

Solving above equation we got

$$g_{NE} = \frac{\mu PC + (S-C)\beta(\mu A - B)}{\mu PC - \beta B(S-C)}$$

where

$$S = \left[1 - \frac{\mu}{2\Delta} ln \frac{\gamma+\Delta}{\gamma-\Delta}\right.$$

$P = \beta A - \gamma\beta\eta(\tau\rho + \rho\omega_2 + \mu\rho + \tau\sigma)$

$A = (\tau + \omega_2 + \mu)(\eta + \mu)$

$B = \alpha\beta SA$

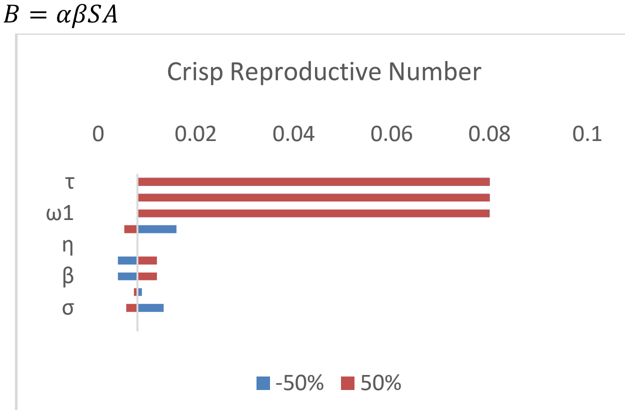

Fig-3: Sensitivity Analysis for response function Crisp Reproductive number

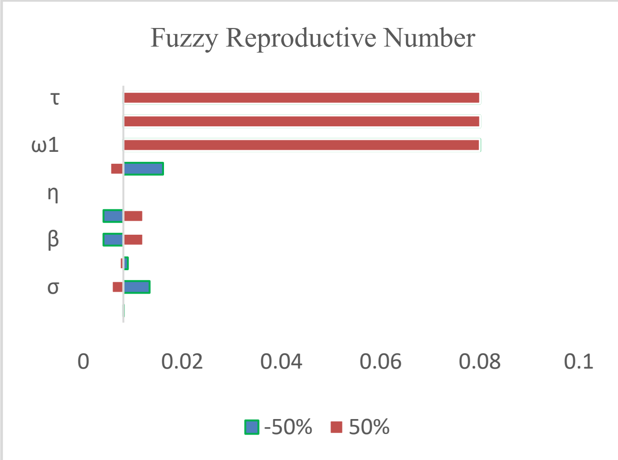

Fig-4: Sensitivity Analysis for response function Fuzzy Reproductive number

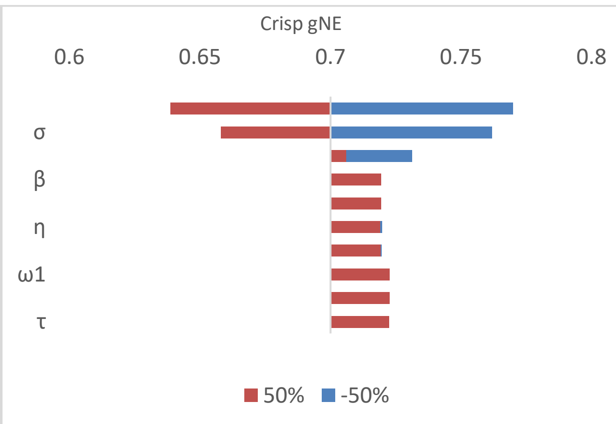

Fig-5: Sensitivity Analysis for response function Cρrisp g_{NE}

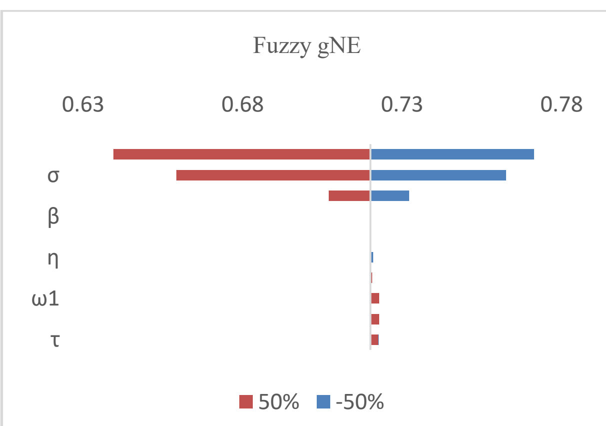

Fig-6: Sensitivity Analysis for response function Fuzzy g_{NE}

4. Conclusion

In this paper we developed a game theory model using triangular fuzzy number. We have considered that the transmission co-efficient is uncertain and we described it by triangular fuzzy number (symmetric). Numerically we compared the crisp model with fuzzy model and studied sensitivity analysis and concluded that if the uncertainties are accounted for in appropriate manner the transmission co-efficient would increase.

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